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# Lepton flavour violation in future linear colliders in the long-lived stau NLSP scenario

#### Alejandro Ibarra

Instituto de Física Teórica, CSIC/UAM, C-XVI Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain E-mail: ibarra@mail.cern.ch

#### Sourov Roy

Helsinki Institute of Physics, University of Helsinki P.O. Box 64, FIN-00014 Finland E-mail: roy@cc.helsinki.fi

ABSTRACT: We analyze the prospects of observing lepton flavour violation in future  $e^-e^-$  and  $e^+e^-$  linear colliders in scenarios where the gravitino is the lightest supersymmetric particle, and the stau is the next-to-lightest supersymmetric particle. The signals consist of multilepton final states with two heavily ionizing charged tracks produced by the long-lived staus. The Standard Model backgrounds are very small and the supersymmetric backgrounds can be kept well under control by the use of suitable kinematical cuts. We discuss in particular the potential of the projected International Linear Collider to discover lepton flavour violation in this class of scenarios, and we compare the estimated sensitivity with the constraints stemming from the non-observation of rare decays.

KEYWORDS: Supergravity Models, Rare Decays, Supersymmetry Phenomenology, Supersymmetric Standard Model.

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#### 1. Introduction

The discovery of flavour violation in neutrino oscillations [1] has opened up a new era for flavour physics in the leptonic sector. This crucial discovery also encourages the search for flavour violation in the charged lepton sector, and if low energy supersymmetry is discovered, in the slepton sector.

The scalar sector of supersymmetric theories contains many new flavour violating couplings, stemming from the off-diagonal elements of the soft breaking terms. Namely, in the mass eigenstate basis for the charged leptons, the soft-breaking Lagrangian reads:

$$-\mathcal{L}_{\text{soft}} = (m_{\widetilde{l}_L}^2)_{ij} \widetilde{l}_{L_i}^{\dagger} \widetilde{l}_{L_j} + (m_{\widetilde{l}_R}^2)_{ij} \widetilde{l}_{R_i}^{\dagger} \widetilde{l}_{R_j} + (Y_i^l A_{lij} \widetilde{l}_{R_i} \widetilde{l}_{L_j} H_d + h.c.), \tag{1.1}$$

where i, j = 1, 2, 3 are generational indices.  $\widetilde{l}_{L,R}$  denote the left and right-handed charged sleptons,  $H_d$  is the down-type Higgs doublet,  $Y_i^l$  is the charged-lepton Yukawa coupling, and  $m_{\widetilde{l}_{L,R}}^2$  and  $A_l$  are the soft scalar mass matrix squared and the soft trilinear matrix, respectively. The resulting  $6 \times 6$  charged-slepton mass matrix reads:

$$\mathcal{M}^{2} = \begin{pmatrix} m_{\tilde{l}_{L}}^{2} + m_{l}^{2} - (\frac{1}{2} - \sin^{2}\theta_{W})m_{Z}^{2}\cos 2\beta & m_{l}(A_{l} - \mu \tan \beta)^{\dagger} \\ m_{l}(A_{l} - \mu \tan \beta) & m_{\tilde{l}_{R}}^{2} + m_{l}^{2} - \sin^{2}\theta_{W}m_{Z}^{2}\cos 2\beta \end{pmatrix}, (1.2)$$

with  $m_l = Y^l \langle H_d^0 \rangle$ . It is customary to express this mass matrix in the form [2]:

$$\mathcal{M}^{2} = \begin{pmatrix} m_{L}^{av2} (\mathbf{1} + \delta_{LL}) & m_{l} (A_{l}^{av} - \mu \tan \beta)^{*} + m_{L}^{av} m_{R}^{av} \delta_{LR} \\ m_{l} (A_{l}^{av} - \mu \tan \beta) + m_{L}^{av} m_{R}^{av} \delta_{RL} & m_{R}^{av2} (\mathbf{1} + \delta_{RR}) \end{pmatrix}, (1.3)$$

so that the amount of flavour violation is parametrized by the  $3 \times 3$  matrices  $\delta_{LL}$ ,  $\delta_{RR}$ ,  $\delta_{LR}$  and  $\delta_{RL}$ , and we assume them to be real in this paper. In this expression,  $m_L^{av}$ ,  $m_R^{av}$  are an average of the masses of the left-handed and right-handed sleptons, respectively, and  $A_l^{av}$  is an average of the diagonal elements of the soft trilinear matrix.

The size of these flavour violating terms is model dependent, although very rarely strictly vanishing [3]. They arise when the mechanism of supersymmetry breaking distinguishes among flavours, thus producing flavour violation in the soft breaking terms already at tree level. For example, in the framework of the Froggatt-Nielsen mechanism [4], the flavon fields not only generate the mass hierarchies and the mixings, but in general contribute to the breaking of supersymmetry, inducing off-diagonal elements in the soft terms [5]. Besides, in weakly coupled string constructions where the Kähler moduli fields participate in the breaking of supersymmetry, if the matter metric is non-diagonal also flavour violating couplings in the scalar sector are generated [6, 7]. Furthermore, even if the supersymmetry breaking mechanism is flavour blind, so that the soft terms are flavour diagonal at the cut-off scale, radiative effects might spoil this diagonal structure. Many well motivated models predict the existence of particles in the desert with flavour violating couplings to the Minimal Supersymmetric Standard Model (MSSM) superfields, that would generate off-diagonal entries in the soft-breaking matrices through quantum corrections. Renowned examples are the MSSM extended with right-handed neutrinos [8] and Grand Unified models [9].

Flavour violation in the scalar sector is propagated through loop effects to the charged lepton sector, inducing rare decays. The non-observation of these processes imposes very strong constraints on the flavour violating couplings, that depend on the particular point of the SUSY parameter space. For future reference, we show in table 1 the constraints on the flavour violating parameters  $\delta_{LL}$ ,  $\delta_{RR}$ ,  $\delta_{LR}$  and  $\delta_{RL}$  for the supersymmetric spectrum of the benchmark point  $\epsilon$  of [10] that follow from the present experimental bounds  $BR(\mu \to e\gamma) \lesssim 1.2 \times 10^{-11}$  [11],  $BR(\tau \to \mu \gamma) < 3.1 \times 10^{-7}$  [12] (Belle) or  $< 6.8 \times 10^{-8}$  [13] (BaBar), and  $BR(\tau \to e\gamma) < 3.9 \times 10^{-7}$  [14]. The relevant formulae to compute the bounds on the  $\delta$ 's from the experimental constraints on the rare decays can be found in the appendix. The  $\epsilon$  benchmark point belongs to the class of scenarios that we would like to study in this paper, namely when the gravitino is the lightest supersymmetric particle and the stau is the next-to-lightest supersymmetric particle, and will be used in this paper to illustrate our results. The complete Higgs and supersymmetric spectra for this benchmark point is summarized in table 2.

The next round of experiments will search for muon and tau rare decays with enhanced sensitivity [15], and will provide improved bounds on the flavour violating couplings, or hopefully, a positive signal for lepton flavour violation. On the other hand, the advent of high energy particle colliders offers new opportunities to study flavour violation. The on-shell production of supersymmetric particles would allow the study of their tree-level flavour violating production and decay. This strategy would offer very valuable information about the scalar sector, that would complement the information provided by rare decays.

The absence of large hadronic backgrounds at  $e^+e^-$  or  $e^-e^-$  colliders makes this class of experiments particularly convenient to study lepton flavour violation [16]. The signals

sector	$\delta_{LL}$	$\delta_{RR}$	$\delta_{LR}$	$\delta_{RL}$
12	$2 \times 10^{-4}$	$6 \times 10^{-4}$	$4 \times 10^{-6}$	$4 \times 10^{-6}$
13	0.09	0.27	0.03	0.03
23	0.04	0.11	0.01	0.01

**Table 1:** Constraints on the flavour violating parameters  $\delta_{LL}$ ,  $\delta_{RR}$ ,  $\delta_{LR}$  and  $\delta_{RL}$ , for the supersymmetric spectrum of the benchmark point  $\epsilon$  of [10] (see details in the text).

$h^0$	119	$\chi_1^0$	183	$\widetilde{e}_L,\widetilde{\mu}_L$	298	$\widetilde{u}_L,\widetilde{c}_L$	897
$H^0$	641	$\chi_2^0$	349	$\widetilde{e}_R,\widetilde{\mu}_R$	169	$\widetilde{u}_R,\widetilde{c}_R$	867
$A^0$	641	$\chi_3^0$	578	$\widetilde{ u}_e,\widetilde{ u}_\mu$	287	$\widetilde{d}_L,\widetilde{s}_L$	901
$H^{\pm}$	646	$\chi_4^0$	593	$\widetilde{ au}_1$	150	$\widetilde{d}_R,\widetilde{s}_R$	864
		1	349	$\widetilde{ au}_2$	302	$\widetilde{t}_1$	682
		$\begin{bmatrix} \chi_1^{\pm} \\ \chi_2^{\pm} \\ \widetilde{g} \end{bmatrix}$	594	$\widetilde{ u}_{ au}$	285	$\widetilde{t}_2 \ \widetilde{b}_1$	879
		$\widetilde{g}$	986			$\widetilde{b}_1$	824
						$\widetilde{b}_2$	862

**Table 2:** Mass spectrum (in GeV) for the benchmark point  $\epsilon$ , taken from table 2 in [10].

will depend crucially on the particular supersymmetric scenario considered. We can differentiate roughly two main classes of scenarios, according to the nature of the lightest supersymmetric particle (LSP). A popular choice for the LSP is the neutralino, although scenarios with superweakly interacting LSP, such as the gravitino or the axino, are also compatible with all the collider experiments and cosmology. In this paper we will concentrate on the latter class of scenarios, focusing for definiteness on the case with gravitino LSP, although the analysis and the conclusions for the case with axino LSP are completely analogous. These scenarios have received a lot of attention recently and their properties have been studied in a number of papers [10, 17–19]. One of the most remarkable features of these scenarios is the longevity of the next-to-lightest supersymmetric particle (NLSP). The NLSP could only decay to gravitinos and Standard Model particles through gravitational interactions, with a decay rate strongly suppressed by the Planck mass. This translates into lifetimes that could be long enough to allow the NLSP to traverse the detector.

Under the assumption of universality of the soft breaking scalar, gaugino and trilinear parameters at some cut-off scale, the NSLP can be either a neutralino or a stau, although in more general scenarios other candidates could also be possible. In the case that the NLSP is the neutralino, the signals for lepton flavour violation will be identical to the case with neutralino LSP, which have been discussed extensively in the literature [20]. On the other hand, the signals for the case with stau NLSP could be very different. Whereas the neutralino escapes detection and is identified as missing energy, the stau would produce a heavily ionizing track in the vertex detector. This signature is very unique, therefore the observation of heavily ionizing tracks would give strong support to this scenario and would allow the search for lepton flavour violation essentially without Standard Model backgrounds.

The longevity of the staus would also allow their collection and the subsequent detection of their decay products with reduced backgrounds. This possibility has been discussed in [10, 17], and in particular the possibility of detecting lepton flavour violation was studied in [19].

This paper is organized as follows. In section II, we present the notation and the setup for our analysis, and discuss the different signatures for lepton flavour violation at future  $e^-e^-$  and  $e^+e^-$  colliders. In section III, we describe in particular the analysis for the case of the projected International Linear Collider (ILC) and estimate the sensitivity reach of this experiment to lepton flavour violation in scenarios where the gravitino is the LSP and the stau the NLSP. We also compare this sensitivity with the present and future constraints on lepton flavour violation stemming from the non-observation of rare leptonic decays. In section IV we present our conclusions, and finally, in the appendix, we include the relevant formulas to compute the branching ratios for the process  $\ell_i \to \ell_j \gamma$ .

### 2. Flavour violating signatures at future electron colliders

Throughout this paper we will assume that the gravitino is the LSP and the NLSP is mainly a right-handed stau, although it could have some admixture of left-handed stau or other leptonic flavours. We will denote the mass eigenstates by the dominant flavour, so that the NLSP will be denoted by  $\tilde{\tau}_1$ . Motivated by the low energy spectrum of the constrained MSSM, we will also assume that next to the  $\tilde{\tau}_1$ , the lightest superparticles are the two combinations of right-handed selectron and smuon, also with a very small admixture of left-handed states (due to the Yukawa suppression of the left-right mixing) and some admixture of stau. The mass splitting between them is expected to be very small, and the absolute values of their masses are expected to be not very different from the NLSP mass. We will denote these states by  $\tilde{e}_R$  and  $\tilde{\mu}_R$ , the former being the mass eigenstate with largest right-handed selectron component and the latter with largest right-handed smuon component. Next in mass in the supersymmetric spectrum are the lightest neutralino and the rest of the sparticles. Schematically the spectrum reads

$$m_{3/2} < m_{\widetilde{\tau}_1} < m_{\widetilde{e}_R, \widetilde{\mu}_R} < m_{\chi_1^0}, m_{\widetilde{e}_L, \widetilde{\mu}_L}, m_{\widetilde{\tau}_2} \dots$$
 (2.1)

It is important for our analysis that the NLSP decays outside the detector, so that it is detected as a heavily ionizing track. If R-parity is conserved, the NLSP can only decay into a gravitino and a charged lepton with total decay rate

$$\Gamma \simeq \frac{m_{\widetilde{\tau}_1}^5}{48\pi m_{3/2}^2 M_P^2} \left(1 - \frac{m_{3/2}^2}{m_{\widetilde{\tau}_1}^2}\right)^4,$$
 (2.2)

where  $M_P = (8\pi G_N)^{-1/2}$  is the reduced Planck mass. Therefore, the requirement that the NLSP decay length is larger than ten centimeters, to guarantee that the NLSP traverses a few layers in the vertex detector leaving a heavily ionizing track, translates into the constraint on the gravitino mass  $m_{3/2} \gg 0.1 \text{ keV} \times (m_{\tilde{\tau}}/100 \text{ GeV})^{5/2}$ .

After discussing the set-up for this analysis, let us discuss the possible signals and backgrounds for the detection of lepton flavour violation. Despite the fact that the discussion is very similar for the  $e^-e^-$  and for the  $e^+e^-$  collider, let us analyze, for the sake of clarity, each case separately.

#### $2.1 e^-e^-$ collider

When lepton flavour is conserved, only left and right-handed selectrons would be produced in this mode, to be precise in the t-channel by neutralino exchange. Since right-handed selectrons are lighter than left-handed selectrons, the largest production cross section would correspond to the process  $e^-e^- \to \widetilde{e_R} \ \widetilde{e_R}$ . The signatures for this process depend crucially on the mass splitting between the right-handed selectron and the NLSP. If the mass splitting is sufficiently large ( $\sim 15$ –20 GeV), the right-handed selectron would decay mainly into charged leptons and a NLSP before reaching the detector, in a process mediated by neutralinos. The signature for this lepton flavour conserving process would be the detection of two heavily ionizing tracks, two electrons and two taus <sup>1</sup>

$$e^- e^- \to \widetilde{e}_R^- \widetilde{e}_R^- \to (e^- \tau^{\pm} \widetilde{\tau}_1^{\mp})(e^- \tau^{\pm} \widetilde{\tau}_1^{\mp}).$$
 (2.3)

If the mass splitting between the right-handed selectron and the NLSP is smaller, the charged leptons could be too soft to be detected, and only the two heavily ionizing tracks would be observed. Finally, if the mass splitting is very small, the decay channel into charged leptons would be kinematically closed. Selectrons could only decay into NLSPs and neutrinos by a process mediated by charginos, with a decay rate suppressed by the small electron Yukawa coupling:

$$e^-e^- \to \widetilde{e}_R^- \widetilde{e}_R^- \to (\nu_e \, \bar{\nu}_\tau \, \widetilde{\tau}_1^-)(\nu_e \, \bar{\nu}_\tau \, \widetilde{\tau}_1^-).$$
 (2.4)

If this is the case, again only two heavily ionizing tracks would be detected, corresponding to the NLSPs or perhaps to the right-handed selectrons, if these are long lived enough to traverse the detector. Therefore, production of two right-handed selectrons would result in the detection of two heavily ionizing tracks, two electrons and two taus; or the detection of just two heavily ionizing tracks.

Less likely than the production of two right-handed selectrons is the associated production of one left-handed selectron and one right-handed selectron. Left-handed selectrons can decay via neutralino exchange either into charged leptons and a NLSP, or via chargino exchange into neutrinos and a NLSP, with comparable decay rates. Therefore, in the detector two heavily ionizing tracks would be observed, with either two electrons and two taus, or one electron and one tau, or no charged leptons:

$$e^- e^- \to \tilde{e}_L^- \tilde{e}_R^- \to (e^- \tau^{\pm} \tilde{\tau}_1^{\mp})(e^- \tau^{\pm} \tilde{\tau}_1^{\mp}),$$
 (2.5)

$$e^- e^- \rightarrow \widetilde{e}_L^- \widetilde{e}_R^- \rightarrow (\nu_e \ \bar{\nu}_\tau \ \widetilde{\tau}_1^-)(e^- \tau^{\pm} \ \widetilde{\tau}_1^{\mp}),$$
 (2.6)

$$e^- e^- \rightarrow \widetilde{e}_L^- \widetilde{e}_R^- \rightarrow (\nu_e \ \bar{\nu}_\tau \ \widetilde{\tau}_1^-)(\nu_e \ \bar{\nu}_\tau \ \widetilde{\tau}_1^-).$$
 (2.7)

<sup>&</sup>lt;sup>1</sup>To be precise, in the final state one could find two taus, two antitaus or a pair of tau-antitau, depending on the charges of the outgoing NLSPs. However, and for simplicity in the notation, we will denote as "tau" either the tau or the antitau, and analogously for the electron and the muon.

(The first two processes would have comparable cross sections, whereas the third would be suppressed by the small electron Yukawa coupling in the decay of  $\widetilde{e}_R^-$ .) Similar signatures would follow from the production of two left-handed selectrons, in this case with comparable cross sections for the three processes. As we will see, processes associated with the production of left-handed selectrons constitute the most important source of background for the detection of lepton flavour violation.

If lepton flavour violation exists in nature at an observable level, novel possibilities arise, namely the associated production of a right-handed selectron and a smuon, or a right-handed selectron and a NLSP. The associated production of a right-handed smuon and a NLSP could also be possible, although it would require two flavour violating vertices and is therefore very suppressed.

The associated production of  $\widetilde{e}_R^-$  and  $\widetilde{\mu}_R^-$  would give rise to the observation of two heavily ionizing tracks, two taus, one muon and one electron:

$$e^- e^- \to \tilde{e}_R^- \tilde{\mu}_R^- \to (e^- \tau^{\pm} \tilde{\tau}_1^{\mp})(\mu^- \tau^{\pm} \tilde{\tau}_1^{\mp}).$$
 (2.8)

No other process in an  $e^-e^-$  collider yields this same signal, and if particle identification is sufficiently good, the observation of this process would represent a clear signal for lepton flavour violation in the selectron-smuon sector, parametrized by the off-diagonal element of the right-handed slepton mass matrix  $(m_{\tilde{l}_R}^2)_{12}$ .

On the other hand, the associated production of  $\widetilde{e_R}$  and  $\widetilde{\tau_1}$  would give rise to the observation of two heavily ionizing tracks (when the mass splitting between the right-handed selectron and the NLSP is small), or two heavily ionizing tracks, one electron and one tau (when the mass splitting is sufficiently large). The former process cannot be distinguished from the production of two right-handed selectrons and cannot be used for the search of lepton flavour violation. However, the latter process could constitute a strong signal of lepton flavour violation:

$$e^- e^- \to \widetilde{e}_R^- \widetilde{\tau}_1^- \to (e^- \tau^{\pm} \widetilde{\tau}_1^{\mp}) \widetilde{\tau}_1^-.$$
 (2.9)

The final state in this process could be mimicked by the production of two right-handed selectrons, and the subsequent decay of one of them into neutrinos and a NLSP by an interaction mediated by higgsinos, while the other decays into charged leptons and a NLSP:  $e^-e^- \to \tilde{e}_R^-\tilde{e}_R^- \to (e^-\tau^+\tilde{\tau}_1^-\tilde{\tau}_1^-\nu_e\bar{\nu}_\tau + e^-\tau^-\tilde{\tau}_1^+\tilde{\tau}_1^-\nu_e\bar{\nu}_\tau)$ . However, the decay mode  $\tilde{e}_R \to \nu_e\bar{\nu}_\tau\tilde{\tau}_1^-$  is highly suppressed due to the presence of the electron Yukawa coupling, which implies decay rates of the order of  $10^{-19}$  GeV. Hence, this source of background can be neglected in general.

A more important background could follow from the associated production of a left-handed and a right-handed selectron, or two left-handed selectrons, where the left-handed selectron decays into neutrinos and a NLSP with unsuppressed decay rate, eq. (2.6). One should note here that these background events include two neutrinos in the final state which give rise to imbalance in momentum. On the other hand, the signal events, eq. (2.9), do not have any neutrinos in the final state and hence there is no missing transverse momentum  $p_T$ . Therefore, if we demand that the final state should have a  $p_T$  < 5 GeV then most

of the background could be eliminated. This background could be further reduced using right-polarized electron beams. In addition, the signal cross section increases with the use of right-polarized beams. Nevertheless, it should be kept in mind that the luminosity is also reduced when one considers polarized beams since only a part of the total integrated luminosity available is used for polarization. In consequence, the signal significance (defined later) in the case of right-polarized beams does not change much compared to that in the case of unpolarized beams and hence the sensitivity of the experiment remains almost unchanged in both cases. Another way to eliminate this background (with a previous knowledge of the spectrum) is to tune the center of mass energy of the collider to be below the threshold for left-handed selectron production.

Additional signals for lepton flavour violation in the right-handed sector follow from the lepton flavour violating decays of the right-handed selectrons. Depending on which particular sector violates lepton flavour, right-handed selectrons could decay into two electrons and a NLSP or two taus and a NLSP, if the violation occurs in the right-handed selectronstau sector, i.e.  $(m_{\tilde{l}_R}^2)_{13} \neq 0$ ; one muon, one tau and a NLSP if it occurs in the right-handed selectron-smuon sector, i.e.  $(m_{\tilde{l}_R}^2)_{12} \neq 0$ ; or one electron, one muon and a NLSP if it occurs in the right-handed smuon-stau sector, i.e.  $(m_{\tilde{l}_R}^2)_{23} \neq 0$ . The corresponding lepton flavour violating processes read:

$$e^- e^- \to \widetilde{e}_R^- \widetilde{e}_R^- \to (e^- e^{\pm} \widetilde{\tau}_1^{\mp}) (e^- \tau^{\pm} \widetilde{\tau}_1^{\mp}) \quad \text{if} \quad (m_{\widetilde{l}_R}^2)_{13} \neq 0,$$
 (2.10)

$$e^{-} e^{-} \to \widetilde{e}_{R}^{-} \widetilde{e}_{R}^{-} \to (e^{-} e^{\pm} \widetilde{\tau}_{1}^{\mp}) (e^{-} \tau^{\pm} \widetilde{\tau}_{1}^{\mp}) \quad \text{if} \quad (m_{\widetilde{l}_{R}}^{2})_{13} \neq 0,$$

$$e^{-} e^{-} \to \widetilde{e}_{R}^{-} \widetilde{e}_{R}^{-} \to (\tau^{-} \tau^{\pm} \widetilde{\tau}_{1}^{\mp}) (e^{-} \tau^{\pm} \widetilde{\tau}_{1}^{\mp}) \quad \text{if} \quad (m_{\widetilde{l}_{R}}^{2})_{13} \neq 0,$$

$$e^{-} e^{-} \to \widetilde{e}_{R}^{-} \widetilde{e}_{R}^{-} \to (\mu^{-} \tau^{\pm} \widetilde{\tau}_{1}^{\mp}) (e^{-} \tau^{\pm} \widetilde{\tau}_{1}^{\mp}) \quad \text{if} \quad (m_{\widetilde{l}_{R}}^{2})_{12} \neq 0,$$

$$(2.10)$$

$$e^- \ e^- \to \widetilde{e}_R^- \ \widetilde{e}_R^- \to (\mu^- \ \tau^{\pm} \ \widetilde{\tau}_1^{\mp}) \ (e^- \ \tau^{\pm} \ \widetilde{\tau}_1^{\mp}) \quad \text{if} \ (m_{\widetilde{l}_R}^2)_{12} \neq 0,$$
 (2.12)

$$e^- e^- \to \tilde{e}_R^- \tilde{e}_R^- \to (e^- \mu^{\pm} \tilde{\tau}_1^{\mp}) (e^- \tau^{\pm} \tilde{\tau}_1^{\mp}) \quad \text{if} \quad (m_{\tilde{l}_B}^2)_{23} \neq 0.$$
 (2.13)

None of these processes suffers from important backgrounds, provided particle identification is sufficiently reliable, and constitute important probes of lepton flavour violation in the right-handed slepton sector.

#### $2.2 e^+ e^-$ collider

Production of sleptons at the  $e^+e^-$  collider proceeds via t-channel by neutralino exchange and via s-channel by photon and Z boson exchange. In the t-channel the possible processes are analogous to those for the  $e^-e^-$  collider, with the appropriate changes in the electric charges of the particles. Namely, when lepton flavour is conserved, only selectrons will be produced, and when lepton flavour is violated, the analogous processes to eqs. (2.8)-(2.13)will occur, with analogous backgrounds.

On the other hand, in the s-channel all types of sleptons can be produced:

$$e^+ e^- \to \tilde{l}_{R_i}^+ \tilde{l}_{R_i}^-,$$
 (2.14)

$$e^+ e^- \to \tilde{l}_{R_i}^{\pm} \tilde{l}_{L_i}^{\mp},$$
 (2.15)

$$e^+ e^- \to \tilde{l}_{L_i}^+ \tilde{l}_{L_j}^-,$$
 (2.16)

where  $\widetilde{l}_i$ ,  $\widetilde{l}_i = \widetilde{e}$ ,  $\widetilde{\mu}$ ,  $\widetilde{\tau}$ .

If lepton flavour is conserved, sleptons would be produced in pairs with opposite lepton family number. The production of a pair of NLSPs with opposite charges would be detected as two back-to-back heavily ionized tracks whereas production of pairs of right-handed smuons or selectrons would be detected as two heavily ionized tracks, together with two taus and two muons or electrons, respectively. Right-handed smuons and selectrons could also decay into neutrinos via higgsino exchange, yielding a signature consisting of just two heavily ionizing tracks plus missing energy. Nevertheless, these processes are very suppressed by the small electron and muon Yukawa couplings and can be usually neglected, even in the large  $\tan \beta$  regime.

The experimental signatures are qualitatively different when left-handed sfermions are produced. These could decay via gaugino exchange either into charged leptons or into neutrinos with comparable decay rates, yielding signatures with two heavily ionizing tracks and four, two or no charged leptons in the final state. For the case of selectron production, the possible final states are

$$e^+ e^- \to \tilde{e}_R^+ \tilde{e}_L^- \to (e^+ \tau^{\pm} \tilde{\tau}_1^{\mp}) (e^- \tau^{\pm} \tilde{\tau}_1^{\mp}),$$
 (2.17)

$$e^+ e^- \to \tilde{e}_R^+ \tilde{e}_L^- \to (e^+ \tau^{\pm} \tilde{\tau}_1^{\mp}) (\nu_e \bar{\nu}_\tau \tilde{\tau}_1^-),$$
 (2.18)

$$e^{+} e^{-} \to \tilde{e}_{L}^{+} \tilde{e}_{L}^{-} \to (e^{+} \tau^{\pm} \tilde{\tau}_{1}^{\mp}) (e^{-} \tau^{\pm} \tilde{\tau}_{1}^{\mp}),$$
 (2.19)

$$e^+ e^- \to \tilde{e}_L^+ \tilde{e}_L^- \to (e^+ \tau^{\pm} \tilde{\tau}_1^{\mp}) (\nu_e \bar{\nu}_\tau \tilde{\tau}_1^-),$$
 (2.20)

$$e^+ e^- \to \widetilde{e}_L^+ \widetilde{e}_L^- \to (\bar{\nu}_e \ \nu_\tau \ \widetilde{\tau}_1^+) \ (\nu_e \ \bar{\nu}_\tau \ \widetilde{\tau}_1^-).$$
 (2.21)

As for the case of the  $e^-e^-$  linear collider, the left-handed slepton decays into neutrinos will represent the most important source of background for the search of lepton flavour violation.

Lepton flavour violation could manifest itself either in the production of sleptons or in their decays. Concentrating just on lepton flavour violation in the right-handed slepton sector, the lepton flavour violating photon or Z-boson vertices would give rise to the following processes:

$$e^+ e^- \to \tilde{e}_R^+ \tilde{\mu}_R^- \to (e^+ \tau^{\pm} \tilde{\tau}_1^{\mp}) (\mu^- \tau^{\pm} \tilde{\tau}_1^{\mp}) \quad \text{if} \quad (m_{\tilde{l}_R}^2)_{12} \neq 0,$$
 (2.22)

$$e^{+} e^{-} \to \widetilde{\mu}_{R}^{+} \widetilde{\tau}_{1}^{-} \to (\mu^{+} \tau^{\pm} \widetilde{\tau}_{1}^{\mp}) \widetilde{\tau}_{1}^{-}$$
 if  $(m_{\widetilde{l}_{R}}^{2})_{23} \neq 0$ , (2.23)

$$e^{+} e^{-} \to \widetilde{\mu}_{R}^{+} \widetilde{\tau}_{1}^{-} \to (\mu^{+} \tau^{\pm} \widetilde{\tau}_{1}^{\mp}) \widetilde{\tau}_{1}^{-} \qquad \text{if } (m_{\widetilde{l}_{R}}^{2})_{23} \neq 0, \qquad (2.23)$$

$$e^{+} e^{-} \to \widetilde{e}_{R}^{+} \widetilde{\tau}_{1}^{-} \to (e^{+} \tau^{\pm} \widetilde{\tau}_{1}^{\mp}) \widetilde{\tau}_{1}^{-} \qquad \text{if } (m_{\widetilde{l}_{R}}^{2})_{13} \neq 0. \qquad (2.24)$$

It is important to note that the lepton flavour violating photon and Z-boson vertices appear only at the one loop level. Consequently the cross sections for these flavour violating processes in the s-channel are suppressed compared to flavour violating processes in the t-channel, and therefore will not be considered in our analysis. Additional flavour violating signatures could stem from the pair production of two neutralinos in the s-channel, followed by the flavour violating decay of one of them into a charged lepton and the NLSP, for example,  $e^+ e^- \to \chi^0 \chi^0 \to (e^{\pm} \widetilde{\tau}_1^{\mp}) (\tau^{\pm} \widetilde{\tau}_1^{\mp})$ . The cross section for this process is also smaller than that of the flavour violating processes proceeding in the t-channel, since in this scenario neutralinos are heavier than sleptons, and will not be considered either.

Lepton flavour violation signals could also stem from the decay of right-handed selectrons, similarly to the case for the  $e^-e^-$  collider, eqs. (2.10)–(2.13):

$$e^+ e^- \to \tilde{e}_R^+ \tilde{e}_R^- \to (e^+ e^\pm \tilde{\tau}_1^\mp) (e^- \tau^\pm \tilde{\tau}_1^\mp) \quad \text{if} \quad (m_{\tilde{l}_R}^2)_{13} \neq 0,$$
 (2.25)

$$e^{+} e^{-} \to \tilde{e}_{R}^{+} \tilde{e}_{R}^{-} \to (\tau^{+} \tau^{\pm} \tilde{\tau}_{1}^{\mp}) (e^{-} \tau^{\pm} \tilde{\tau}_{1}^{\mp}) \quad \text{if} \quad (m_{\tilde{t}_{R}}^{2})_{13} \neq 0,$$
 (2.26)

$$e^{+} e^{-} \to \tilde{e}_{R}^{+} \tilde{e}_{R}^{-} \to (\mu^{+} \tau^{\pm} \tilde{\tau}_{1}^{\mp}) (e^{-} \tau^{\pm} \tilde{\tau}_{1}^{\mp}) \quad \text{if} \quad (m_{\tilde{l}_{R}}^{2})_{12} \neq 0,$$
 (2.27)

$$e^{+} e^{-} \to \tilde{e}_{R}^{+} \tilde{e}_{R}^{-} \to (e^{+} \mu^{\pm} \tilde{\tau}_{1}^{\mp}) (e^{-} \tau^{\pm} \tilde{\tau}_{1}^{\mp}) \quad \text{if} \quad (m_{\tilde{l}_{R}}^{2})_{23} \neq 0,$$
 (2.28)

and analogous processes when it is  $\widetilde{e}_R^-$  the particle that decays violating flavour instead of  $\widetilde{e}_R^+$ . On the other hand, production and lepton flavour violating decay of right-handed smuons would yield the following signals:

$$e^+ e^- \to \widetilde{\mu}_R^+ \widetilde{\mu}_R^- \to (\mu^+ \mu^{\pm} \widetilde{\tau}_1^{\mp}) (\mu^- \tau^{\pm} \widetilde{\tau}_1^{\mp}) \quad \text{if} \quad (m_{\widetilde{l}_R}^2)_{23} \neq 0,$$
 (2.29)

$$e^+ e^- \to \widetilde{\mu}_R^+ \widetilde{\mu}_R^- \to (\tau^+ \tau^{\pm} \widetilde{\tau}_1^{\mp}) (\mu^- \tau^{\pm} \widetilde{\tau}_1^{\mp}) \quad \text{if} \quad (m_{\widetilde{t}_R}^2)_{23} \neq 0,$$
 (2.30)

$$e^+ e^- \to \widetilde{\mu}_R^+ \widetilde{\mu}_R^- \to (e^+ \tau^{\pm} \widetilde{\tau}_1^{\mp}) (\mu^- \tau^{\pm} \widetilde{\tau}_1^{\mp}) \quad \text{if} \quad (m_{\widetilde{l}_B}^2)_{12} \neq 0,$$
 (2.31)

$$e^+ e^- \to \widetilde{\mu}_R^+ \widetilde{\mu}_R^- \to (\mu^+ e^\pm \widetilde{\tau}_1^\mp) (\mu^- \tau^\pm \widetilde{\tau}_1^\mp) \quad \text{if} \quad (m_{\widetilde{\ell}_R}^2)_{13} \neq 0,$$
 (2.32)

and analogously when  $\widetilde{\mu}_R^-$  decays violating flavour instead of  $\widetilde{\mu}_R^+$ . No other process in the  $e^+e^-$  collider yields the same signals, therefore, if particle identification is sufficiently good, the observation of these processes would constitute robust evidences for lepton flavour violation in the right-handed slepton sector.

## 3. Discovery potential of the ILC

In this section we analyze the prospects to observe lepton flavour violation in the projected International Linear Collider (ILC), both for the  $e^-e^-$  mode and the  $e^+e^-$  mode. For definiteness, we will assume a center of mass energy  $\sqrt{s} = 500 \,\text{GeV}$ , and an integrated luminosity of  $500 \,\text{fb}^{-1}$ . We will also assume in our numerical analyses that the beams are unpolarized, although as discussed in the previous section, the use of polarized beams would enhance the strength of the lepton flavour violating signals and reduce the background cross section. The cross sections of the signal events have been calculated in the narrow width approximation. We have calculated the relevant  $2\rightarrow 2$  differential cross section  $d\sigma/d\cos\theta$  and then folded into it the appropriate branching fractions of the corresponding decay channels to get the various final states described earlier.

We select the signal events according to the following criteria:

<sup>&</sup>lt;sup>2</sup>The International Linear Collider is likely to operate in two phases; the first with  $\sqrt{s} = 500 \,\text{GeV}$  and the second with  $\sqrt{s} = 1 \,\text{TeV}$ , with an integrated luminosity of  $1 \,\text{ab}^{-1}$  for the  $e^+e^-$  mode, and smaller for the  $e^-e^-$  mode [21].

<sup>&</sup>lt;sup>3</sup>The projected International Linear Collider is expected to achieve the projected luminosity of 500 fb<sup>-1</sup> with at least an 80% electron polarization and a 60% positron polarization at the interaction point. A degree of polarization of a 90% for the electrons and a 75% for the positrons could possibly be achieved at the cost of some reduction in luminosity [22].

- The transverse momentum of the electrons, the positrons and  $\mu^{\pm}$  must be large enough:  $p_T^{e^{\pm},\mu^{\pm}} > 5 \,\text{GeV}$ .
- Slightly stronger selection criterion has been set on the transverse momentum of the  $\tau$ s:  $p_T^{\tau} > 10 \,\text{GeV}$ .
- The transverse momentum of the  $\tilde{\tau}_1$ s must satisfy  $p_T^{\tilde{\tau}_1} > 10 \, \text{GeV}$ .
- The electrons, the positrons, the  $\mu^{\pm}$  and the  $\tau$ s and both the staus must be relatively central, *i.e.* their pseudorapidities must fall in the range  $|\eta^{e^{\mp},\mu^{\pm},\tilde{\tau}_1,\tau}| < 2.5$ .
- The electrons, the positrons, the  $\mu^{\pm}$ , the  $\tau$ s and the staus must be well-separated from each other: *i.e.* the isolation variable  $\Delta R \equiv \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2}$  (where  $\eta$  and  $\phi$  denote the separation in rapidity and the azimuthal angle, respectively) should satisfy  $\Delta R > 0.4$  for each combination.
- The missing transverse momentum  $p_T < 5 \,\text{GeV}$ .

To illustrate the discovery potential of lepton flavour violation in scenarios with stau NLSP at the ILC we will show contour plots of constant cross section for a variant of the  $\epsilon$  benchmark point of [10]. In our study we will take all the supersymmetric parameters as in the  $\epsilon$  benchmark point, but we will vary the NLSP mass between 144 GeV and 167 GeV (recall that in this benchmark point the mass of the next-to-NLSP, the right-handed selectron, is 169 GeV). We will also admit some small amount of lepton flavour violation in the right-handed slepton sector, parametrized by  $\delta_{RR}^{ij}$ .

At this stage it is very important to discuss the efficiency of identification of the two approximately back-to-back stau tracks from muon tracks. For sufficiently long-lived staus, there are two traditional ways of identification of slowly moving charged massive particles: (1) using a time-of-flight (ToF) device and (2) measuring the associated high ionization energy loss rates dE/dx in the vertex detector and tracking chambers. In the case of a time-of-flight device, one compares the mean time of flight for a muon ( $\beta \simeq 1$ ) to that of the long-lived massive charged particle ( $\beta < 1$ ) for a flight path length of the order of a few meters. Considering a future linear collider operating with 1.4 ns of bunch separation and a detector capable to measure the time of flight with a 50 ps error, it has been proved possible to identify the back-to-back tracks as long-lived staus, with an efficiency varying between  $\sim 60-80\%$  in the mass range relevant for our scenario, 140–170 GeV [23].

An alternative possibility to identify the staus is the measurement of the rate of energy loss, dE/dx, that depends on the  $\beta\gamma$  of the particle. It has been shown in ref. [23], that assuming a 5% resolution in the measurement of dE/dx and using suitable cuts, it could be possible to identify the staus. However, the identification efficiency decreases significantly for masses around 150 GeV, which are precisely the masses relevant for our study.<sup>4</sup> Therefore, we will assume for our study that the staus can be identified just with

<sup>&</sup>lt;sup>4</sup>In this mass range the stau has  $\beta\gamma \sim 1.3$  for a  $\sqrt{s} = 500\,\mathrm{GeV}$  linear collider, and the energy deposited coincides with the one by the muon (with  $\beta\gamma \sim \mathcal{O}(10^3)$ ), hence the drastic reduction in the identification efficiency.

a ToF device and we will use for the efficiency a conservative value of a 60%. We will multiply the cross section of the signal events with this efficiency in order to get realistic numbers. A more detailed analysis on this issue is beyond the scope of this study.

## 3.1 $e^-e^-$ collider

As discussed in previous sections, in the  $e^-e^-$  mode there are mainly two channels where lepton flavour violation in the right-handed selectron-stau sector could be discovered, namely processes with lepton flavour violating production of sleptons:

$$e^-e^- \to \widetilde{e}_R^- \widetilde{\tau}_1^- \to (e^- \tau^{\pm} \widetilde{\tau}_1^{\mp}) \widetilde{\tau}_1^-,$$
 (3.1)

or with lepton flavour violating decay of selectrons:

$$e^- e^- \to \tilde{e}_R^- \tilde{e}_R^- \to (e^- e^{\pm} \tilde{\tau}_1^{\mp}) (e^- \tau^{\pm} \tilde{\tau}_1^{\mp}),$$
 (3.2)

$$e^- e^- \to \tilde{e}_R^- \tilde{e}_R^- \to (\tau^- \tau^{\pm} \tilde{\tau}_1^{\mp}) (e^- \tau^{\pm} \tilde{\tau}_1^{\mp}).$$
 (3.3)

In figure 1 we show contours of constant cross section for the process with lepton flavour violating production of sleptons, that manifest itself as two heavily ionizing tracks and two charged leptons. We show the results in the  $\delta_{RR}^{13}$ - $\Delta m$  plane with the cuts mentioned above (here,  $\Delta m \equiv m_{\tilde{e}_R} - m_{\tilde{\tau}_1}$ .) As discussed in the text, this process suffers from backgrounds stemming from the associated production of  $\tilde{e}_L \tilde{e}_R$  and the subsequent decay of the left-handed selectron into neutrinos. Nevertheless, imposing our cuts these background processes have very small cross sections and can be neglected in general, as can be realized from figure 2, where we show an upper bound on the background cross section, calculated assuming for simplicity  $BR(\tilde{e}_L^- \to \nu_e \bar{\nu}_\tau \tilde{\tau}_1^-) = 1$ .

Demanding that the signal significance (defined as  $S/\sqrt{S+B}\approx \sqrt{S}$ , where S is the number of signal events and B is the number of background events) is larger than 5, we find that searching for this final state in the  $e^-e^-$  mode, the ILC could be sensitive to lepton flavour violation down to the level  $\delta_{RR}^{13}\sim 0.02$ , when the supersymmetric spectrum is as in the  $\epsilon$  benchmark point (i.e. with a mass splitting between the right-handed selectron and the NLSP of  $\sim 20\,\mathrm{GeV}$ ). The sensitivity would improve when the mass splitting increases, whereas for smaller mass splittings, the outgoing charged leptons would be too soft and the cross section would be significantly reduced. For the  $\epsilon$  benchmark point, the sensitivity of the ILC to lepton flavour violation is better than the present sensitivity of experiments searching for the rare decay  $\tau \to e\gamma$ ,  $\delta_{RR}^{13} \sim 0.27$  (cf. table I) and comparable to the projected sensitivity of present B-factories,  $\delta_{RR}^{13} \sim 0.04$ , that follows from the projected bound  $BR(\tau \to e\gamma) \lesssim 10^{-8} \, [24].^5$  On the other hand, future super-B factories could produce of the order of  $10^{10} \, \tau$  pairs at a luminosity of  $10 \, ab^{-1}$ , allowing to probe branching ratios for the rare  $\tau$  decays down to the level of  $10^{-8} - 10^{-9} \, [25]$ , which would translate into a sensitivity reach of  $\delta_{RR}^{13} \sim 0.01$ .

We have seen from the above discussion that the future sensitivity to lepton flavour violation in rare decay experiments is comparable to that at the ILC in the  $\epsilon$  benchmark

<sup>&</sup>lt;sup>5</sup>The bounds on  $\delta_{RR}^{ij}$  roughly scale with  $BR(l_j \to l_i \gamma)^{1/2}$ .

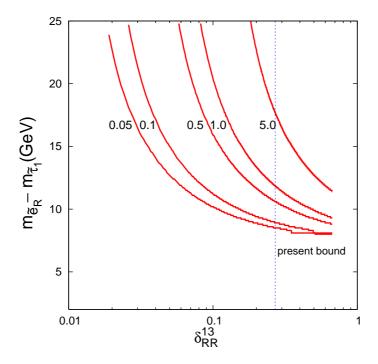


Figure 1: Contours of constant  $\sigma(e^-e^- \to \tilde{e}_R\tilde{\tau}_1^- \to e^-\tau^+\tilde{\tau}_1^-\tilde{\tau}_1^- + e^-\tau^-\tilde{\tau}_1^+\tilde{\tau}_1^-)$  in fb with  $\sqrt{s} = 500 \,\text{GeV}$ , and the present experimental upper bound on  $\delta^{13}$  coming from the non-observation of the process  $\tau \to e\gamma$ . The remaining parameters of the model are chosen as in the  $\epsilon$  point (see text for details). Both  $e^-$  beams are unpolarized.

point. However, there could be other regions in the parameter space where the decay rate for  $\tau \to e\gamma$  is suppressed. This could be due to cancellations in the loops, whereas the tree-level lepton flavour violating production of sleptons may still remain unsuppressed. Furthermore, the observation of rare decays does not shed any light on the source of lepton flavour violation: whether it is in the left-handed sector, the right-handed sector or the trilinear soft terms. On the other hand, the observation of the process  $e^-e^- \rightarrow$  $(e^- \tau^{\pm} \widetilde{\tau}_1^{\mp}) \widetilde{\tau}_1^-$  at the linear collider would pinpoint the right-handed sector as one of the sources of lepton flavour violation. Complementing this information with the one from rare decays could help to identify the sources of flavour violation in the leptonic sector. This could provide invaluable information about the soft-breaking Lagrangian. To be more precise, assuming that the process  $e^ e^- \to (e^ \tau^{\pm}$   $\widetilde{\tau}_1^{\mp})$   $\widetilde{\tau}_1^-$  is observed at the ILC, the quantity  $\delta_{RR}^{13}$  inferred from experiments could be used to predict the rate for  $\tau \to e\gamma$ . If the observed rate for this rare decay is larger than the predicted one, it would follow that there are necessarily additional sources of flavour violation in the leptonic Lagrangian (either in  $\delta_{LL}^{13}$ ,  $\delta_{RL}^{13}$  or  $\delta_{LR}^{13}$ ). If these rates are comparable, it would follow that the right-handed sector is the dominant source of lepton flavour violation; and if the observed rate is smaller,

it would follow that different contributions to the decay amplitude are canceling each other in order to produce a suppressed decay rate.

In this class of scenarios, lepton flavour violation could also be studied at future colliders using stopped staus [17]. At the LHC, cascade decays of squarks and gluinos could produce of the order of 10<sup>6</sup> NLSPs per year if particle masses are close to the current experimental limit. A fraction of them would be stopped in the walls of the detector, <sup>6</sup> and decay at late times producing very energetic particles that would spring from inside the detector. If there is no LFV, all the NLSPs would decay into taus and gravitinos,  $\widetilde{\tau} \to \tau \psi_{3/2}$ . If on the contrary LFV exists in nature, some of the NLSPs would decay into electrons and muons. Therefore the detection of very energetic particles coming from inside the detector would constitute a signal of lepton flavour violation. Using this technique, it has been estimated that at the LHC or the ILC it would be possible to probe down to  $\delta_{RR}^{13} \sim 3 \times 10^{-2} \ (9 \times 10^{-3}) \ \text{if} \ 3 \times 10^3 \ (3 \times 10^4) \ \text{staus could be collected} \ [19].$ 

On the other hand, in figure 3 we consider the lepton flavour violating signals in the decays of the right-handed selectrons, eqs. (3.2), (3.3), that can be detected as two heavily ionizing tracks and four charged leptons. We see from these two figures that the lepton flavour violating signal from  $e^ e^ \to$   $\widetilde{e}_R^ \widetilde{e}_R^ \to$   $(e^ e^\pm$   $\widetilde{\tau}_1^\mp)$   $(e^ \tau^\pm$   $\widetilde{\tau}_1^\mp)$  could be used to probe a larger region in the relevant parameter space compared to that from  $e^- \ e^- \to \widetilde{e_R} \ \widetilde{e_R} \to (\tau^- \ \tau^{\pm} \ \widetilde{\tau}_1^{\mp}) \ (e^- \ \tau^{\pm} \ \widetilde{\tau}_1^{\mp}).$  This is due to the fact that the final states in these two processes are different and we have used different  $p_T$  cuts on the  $\tau$ s and the electrons (positron).

Although the source of lepton flavour violation is  $\delta_{RR}^{13}$  in both figures (1) and (3), the region in the parameter space explored by the process in figure (1) is larger than that in figures (3(a)) or (3(b)). There are two reasons for this feature. In the case when the lepton flavour violation is in the decays of the right selectron, due to the presence of  $(m_{\widetilde{I}_{R}}^{2})_{13}$  there is a branching ratio suppression in different channels shown in eq. (2.10) and in eq. (2.11). Also, since the final states are different when one considers lepton flavour violation in production, the effects of the cuts are also different.

Other lepton flavour violating decays of the right-handed selectron which could be discovered in an  $e^-e^-$  collider are:

$$e^-e^- \to \widetilde{e}_R^- \widetilde{e}_R^- \to (\mu^- \tau^{\pm} \widetilde{\tau}_1^{\mp}) \ (e^- \tau^{\pm} \widetilde{\tau}_1^{\mp}) \quad \text{if} \ (m_{\widetilde{l}_R}^2)_{12} \neq 0,$$
 (3.4)

$$e^{-} e^{-} \to \widetilde{e}_{R}^{-} \widetilde{e}_{R}^{-} \to (\mu^{-} \tau^{\pm} \widetilde{\tau}_{1}^{\mp}) (e^{-} \tau^{\pm} \widetilde{\tau}_{1}^{\mp}) \quad \text{if} \quad (m_{\widetilde{l}_{R}}^{2})_{12} \neq 0,$$

$$e^{-} e^{-} \to \widetilde{e}_{R}^{-} \widetilde{e}_{R}^{-} \to (e^{-} \mu^{\pm} \widetilde{\tau}_{1}^{\mp}) (e^{-} \tau^{\pm} \widetilde{\tau}_{1}^{\mp}) \quad \text{if} \quad (m_{\widetilde{l}_{R}}^{2})_{23} \neq 0.$$

$$(3.4)$$

In figures (4a) and (4b), we have shown contours of constant cross sections of these two processes in the plane  $(\delta_{RR}^{12} - \Delta m)$  for the process in eq. (3.4) and similarly in the plane  $(\delta_{RR}^{23} - \Delta m)$  for the process in eq. (3.5). Obviously, the reach in the process in eq. (3.5) is larger since it contains only one  $\tau$  and hence is less affected by the  $p_T$  cuts. On the other hand, if one compares the process in eq. (3.2) with the process in eq. (3.5) then one can observe that though these two processes look very similar (since we impose similar  $p_T$  cuts

<sup>&</sup>lt;sup>6</sup>A larger number of staus could be trapped by placing 1-10 kton massive material around the LHC detectors, to be precise around  $\mathcal{O}(10^3 - 10^4)$ . Similarly, at the ILC up to  $\mathcal{O}(10^3 - 10^5)$  could be collected and studied.

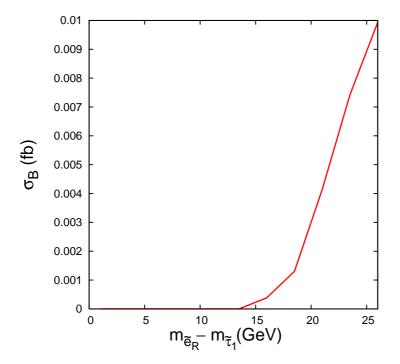
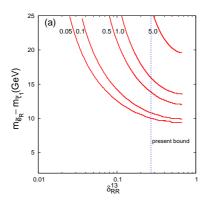


Figure 2: Upper bound on the background cross section (as discussed in the text)  $\sigma_B$  in fb in an  $e^-e^-$  collider, with  $\sqrt{s} = 500\,\mathrm{GeV}$ , as a function of the mass-difference between the right-handed selectron and the NLSP. The remaining parameters of the model are chosen as in the  $\epsilon$  point (see text for details). Both  $e^-$  beams are unpolarized.

for the  $e^{\pm}$  and the  $\mu^{\pm}$ ), the behaviours are different for higher values of the corresponding parameters  $\delta_{RR}^{ij}$  and  $\Delta m$ . This is again due to the fact that there is a branching ratio suppression in the process in eq. (3.2) for relatively large values of the  $\delta_{RR}^{13}$  and  $\Delta m$ . For very low values of  $\delta_{RR}^{ij}$  the nature is very similar in both the figures. Again, for low values of  $\Delta m$  and large values of  $\delta_{RR}^{ij}$  these two figures show quite similar behaviour since the branching ratio suppression in the process in eq. (3.2) is less pronounced in this region and the lower limits in  $\Delta m$  are determined by the  $p_T$  cuts employed.

As before, if we demand that the signal significance is greater than or equal to 5, for the  $\epsilon$  benchmark point the ILC could probe lepton flavour violation down to the level  $\delta_{RR}^{12} \sim 0.04$ ,  $\delta_{RR}^{23} \sim 0.03$ , i.e. for a mass splitting between the right-handed selectron and the NLSP of  $\sim 20$  GeV. For this benchmark point, the sensitivity to lepton flavour violation in the smuon-stau sector is slightly better than the present sensitivity from the rare decay  $\tau \to \mu \gamma$ , and in the smuon-selectron sector, is much worse than from the decay  $\mu \to e \gamma$  (cf. table I).

For the sake of completeness, let us now discuss the process  $e^ e^ \to$   $\widetilde{e}_R^ \widetilde{\mu}_R^ \to$   $(e^ \tau^\pm$   $\widetilde{\tau}_1^\mp)(\mu^ \tau^\pm$   $\widetilde{\tau}_1^\mp)$ . The cross section for this process should be slightly smaller than



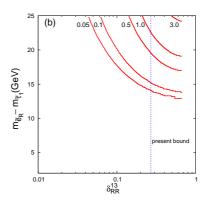
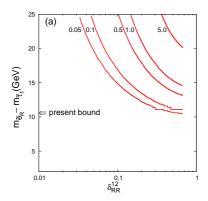


Figure 3: Contours of constant (a)  $\sigma(e^-e^- \to \tilde{e}_R\tilde{e}_R \to e^-e^-e^\pm\tau^\pm\tilde{\tau}_1^\mp\tilde{\tau}_1^\mp)$  and (b)  $\sigma(e^-e^- \to \tilde{e}_R\tilde{e}_R \to \tau^-e^-\tau^\pm\tilde{\tau}_1^\mp\tilde{\tau}_1^\mp)$  in fb with  $\sqrt{s}=500\,\mathrm{GeV}$ , and the present experimental upper bound on  $\delta^{13}$  coming from the non-observation of the process  $\tau\to e\gamma$ . The remaining parameters of the model are chosen as in the  $\epsilon$  point (see text for details). Both  $e^-$  beams are unpolarized.



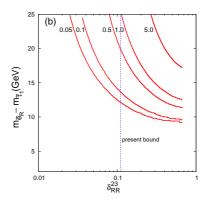


Figure 4: Contours of constant (a)  $\sigma(e^-e^- \to \tilde{e}_R\tilde{e}_R \to \mu^-e^-\tau^\pm\tau^\pm\tilde{\tau}_1^\mp\tilde{\tau}_1^\mp)$  and (b)  $\sigma(e^-e^- \to \tilde{e}_R\tilde{e}_R \to e^-e^-\mu^\pm\tau^\pm\tilde{\tau}_1^\mp\tilde{\tau}_1^\mp)$  in fb with  $\sqrt{s}=500\,\mathrm{GeV}$ , and the present experimental upper bounds on  $\delta^{12}$  and  $\delta^{23}$  coming from the non-observation of the processes  $\mu\to e\gamma$  and  $\tau\to\mu\gamma$ , respectively. Note that the present upper bound for  $\delta^{12}$  lies outside figure (a). The remaining parameters of the model are chosen as in the  $\epsilon$  point (see text for details). Both  $e^-$  beams are unpolarized.

for the process  $e^-e^- \to \widetilde{e}_R^- \widetilde{\tau}_1^- \to (e^-\tau^\pm \widetilde{\tau}_1^\mp) \widetilde{\tau}_1^-$ , due to the larger multiplicity of the final state of the former process, that translates into a bigger impact of the kinematical cuts and the reduction of the signal strength. Thus, the sensitivity to  $\delta_{RR}^{12}$  through the observation of the process with associated production of a right-handed selectron and a right-handed smuon, eq. (2.8), should be slightly smaller than the sensitivity to  $\delta_{RR}^{13}$  coming from the associated production of a right-handed selectron and a NLSP, eq. (2.9). On the other

hand, the final state for the process with associated production of a right-handed selectron and a right-handed smuon, eq. (2.8), is the same as the final state for the pair production of two right-handed selectrons, followed by the flavour violating decay of one of them into a muon, a tau and a NLSP, eq. (2.12). If we combine the cross sections of these two processes to search for lepton flavour violation, the sensitivity to  $\delta_{RR}^{12}$  will increase significantly.

From the above discussion we can see that different regions in the corresponding parameter space can be explored in the lepton flavour violating production and decays of the right-selectron in a future  $e^-e^-$  collider.

## 3.2 $e^+ e^-$ collider

Next, we will discuss the situation in an  $e^+e^-$  collider. As discussed in section IIB, lepton flavour violating production processes are similar to the ones in the  $e^-e^-$  collider with the corresponding changes in the electric charge of the particles. We would also like to reiterate that only the t-channel neutralino mediated diagrams are taken into account for the calculation of these production processes, since the s-channel contributions through photon and Z-boson exchange are loop-suppressed. Let us first consider the case when only  $\delta_{RR}^{13}$  is non-vanishing. In this scenario the lepton flavour violating production process in an  $e^+e^-$  collider looks like

$$e^+ e^- \to \tilde{e}_R^+ \tilde{\tau}_1^- + \tilde{e}_R^- \tilde{\tau}_1^+ \to (e^+ \tau^{\pm} \tilde{\tau}_1^{\mp}) \tilde{\tau}_1^- + (e^- \tau^{\pm} \tilde{\tau}_1^{\mp}) \tilde{\tau}_1^+.$$
 (3.6)

The contours of constant cross sections of the process in eq. (3.6) have been plotted in figure 5 in the  $(\delta_{RR}^{13} - \Delta m)$  plane for other parameter choices as in figure 1 and with  $\sqrt{s} = 500 \, \text{GeV}$ . The background to this process can come from the associated production  $e^+e^- \to \widetilde{e}_R^{\pm} \widetilde{e}_L^{\mp}$  as shown in eq. (2.18) (with the corresponding modifications for the charge conjugate process). The cross section of the background process is plotted in figure 6 as a function of the mass difference between the right-selectron and the NLSP. Comparing figure 1 and figure 5 and looking at the background cross section we observe that the parameter region which can be explored in an  $e^+e^-$  collider with  $5\sigma$  significance is slightly smaller than in the case of an  $e^-e^-$  collider, to be precise,  $\delta_{RR}^{13} \gtrsim 0.03$  against  $\delta_{RR}^{13} \gtrsim 0.02$ for  $\Delta m \lesssim 20 \, \text{GeV}$ . This is due to the fact that the production cross section of the lepton flavour violating process in an  $e^+e^-$  collider is smaller compared to the corresponding process in an  $e^-e^-$  collider. On the other hand, the smaller cross sections in the  $e^+e^-$  for the lepton flavour violating processes could be compensated with a bigger luminosity, as will presumably occur since the electrons in the  $e^-e^-$  collider repel each other translating into a decrease in the luminosity with respect to the  $e^+e^-$  mode. Also, using right-polarized electron and positron beams the signal strength could be enhanced and the background cross section can be reduced further. However, as discussed earlier the sensitivity to lepton flavour violation does not change significantly because of the reduction in luminosity for polarized beams.

In figure 7, we consider lepton flavour violation in right-handed selectron decays as in the case of an  $e^-e^-$  collider. These signals are generated from the  $\tilde{e}_R^+\tilde{e}_R^-$  pair production followed by their lepton flavour violating decays and can be detected as four charged leptons

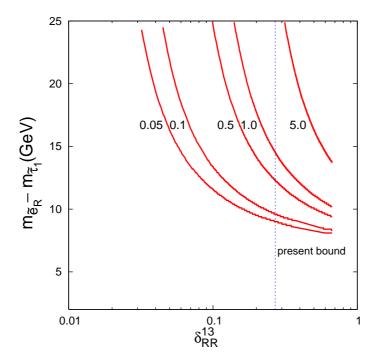


Figure 5: Contours of constant cross sections  $\sigma(e^+e^- \to \tilde{e}_R^+\tilde{\tau}_1^- + \tilde{e}_R^-\tilde{\tau}_1^+ \to e^+\tau^\mp\tilde{\tau}_1^\pm\tilde{\tau}_1^- + e^-\tau^\mp\tilde{\tau}_1^\pm\tilde{\tau}_1^+)$ in fb with  $\sqrt{s} = 500 \, \text{GeV}$ , and the present experimental upper bound on  $\delta^{13}$  coming from the nonobservation of the process  $\tau \to e\gamma$ . The remaining parameters of the model are chosen as in the  $\epsilon$ point (see text for details). Both the  $e^+$  and the  $e^-$  beam are unpolarized.

and two heavily ionizing charged tracks. Looking at figure 7(b) and figure 5, we see that the process  $e^+ e^- \to \tilde{e}_R^+ \tilde{e}_R^- \to (\tau^+ \tau^{\pm} \tilde{\tau}_1^{\mp}) (e^- \tau^{\pm} \tilde{\tau}_1^{\mp})$  is not an efficient way to search for lepton flavour violation, compared to the process with two charged leptons in the final state. On the other hand, figure 7(a) shows that the sensitivity of the  $e^+e^-$  collider for the lepton flavour violating process  $e^+$   $e^- \to \widetilde{e}_R^+$   $\widetilde{e}_R^- \to (e^+$   $e^\pm$   $\widetilde{\tau}_1^\mp)$   $(e^ \tau^\pm$   $\widetilde{\tau}_1^\mp)$  is better than the present sensitivity of the experiments,  $\delta_{RR}^{13} \sim 0.27$  when  $\Delta m \sim 20 \, {\rm GeV}$ , although not as good as the corresponding sensitivity in an  $e^-e^-$  collider (see figure 3). This is again because of the fact that the right-selectron pair production cross section is larger in an  $e^-e^-$  collider since both t- and u-channel diagrams are present and they interfere constructively.

In order to find out the sensitivity to  $\delta_{RR}^{12}$  and  $\delta_{RR}^{23}$  in an  $e^+e^-$  collider, we have plotted in figure 8 the contours of constant cross sections for the processes

$$e^{+} e^{-} \to \tilde{e}_{R}^{+} \tilde{e}_{R}^{-} \to (\mu^{+} \tau^{\pm} \tilde{\tau}_{1}^{\mp}) (e^{-} \tau^{\pm} \tilde{\tau}_{1}^{\mp}),$$

$$e^{+} e^{-} \to \tilde{e}_{R}^{+} \tilde{e}_{R}^{-} \to (e^{+} \mu^{\pm} \tilde{\tau}_{1}^{\mp}) (e^{-} \tau^{\pm} \tilde{\tau}_{1}^{\mp}),$$

$$(3.7)$$

$$e^{+} e^{-} \to \tilde{e}_{R}^{+} \tilde{e}_{R}^{-} \to (e^{+} \mu^{\pm} \tilde{\tau}_{1}^{\mp}) (e^{-} \tau^{\pm} \tilde{\tau}_{1}^{\mp}),$$
 (3.8)

respectively. Demanding that the signal significance is greater or equal to 5, we obtain

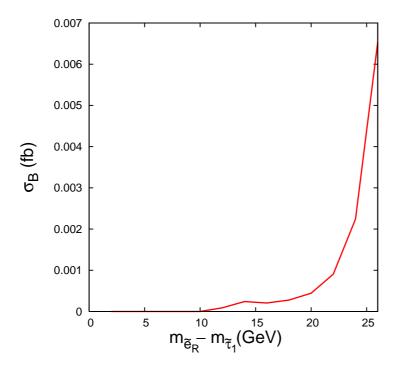
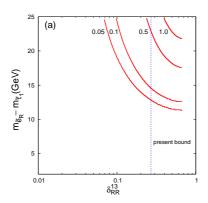


Figure 6: Upper bound on the background cross section (as discussed in the text)  $\sigma_B$  in fb in a  $e^+e^-$  collider, with  $\sqrt{s}=500\,\mathrm{GeV}$ , as a function of the mass-difference between the right-handed selectron and the NLSP. The remaining parameters of the model are chosen as in the  $\epsilon$  point (see text for details). Both the  $e^+$  and  $e^-$  beams are unpolarized.

that the ILC in the  $e^+e^-$  mode could be sensitive to  $\delta_{RR}^{12}\sim 0.1$  and  $\delta_{RR}^{23}\sim 0.08$  when  $\Delta m\sim 20\,{\rm GeV}$ . Once again, the sensitivity to these flavour violating quantities are slightly poorer than at the  $e^-e^-$  collider.

As we also argued in the case of  $e^-e^-$  collider, the process  $e^+$   $e^- \to \widetilde{e}_R^+$   $\widetilde{\mu}_R^- \to (e^+ \ \tau^\pm \ \widetilde{\tau}_1^\mp) \ (\mu^- \ \tau^\pm \ \widetilde{\tau}_1^\mp)$  should have a cross section slightly smaller than the process  $e^+ \ e^- \to \widetilde{e}_R^+ \ \widetilde{\tau}_1^- \to (e^+ \ \tau^\pm \ \widetilde{\tau}_1^\mp) \ \widetilde{\tau}_1^-$  and hence the sensitivity to  $\delta_{RR}^{12}$  through the observation of the former process is slightly poorer than the sensitivity to  $\delta_{RR}^{13}$  obtained from the observation of the latter. However, as pointed out in the  $e^-e^-$  case, the final state of the process with associated production of a right-handed selectron and a right-handed smuon is identical to that in eq. (3.7) and combining the two cross sections should improve the sensitivity of  $\delta_{RR}^{12}$  to a significant extent.

Finally, let us discuss briefly the processes with pair production of two right-handed smuons, followed by the lepton flavour violating decay of one of them, eqs. (2.29)–(2.32). The production cross section of a pair of right-handed smuons is suppressed approximately by a factor of 5 compared to the right-handed selectron pair production. In consequence, the constraints on the different  $\delta_{RR}^{ij}$  from the non-observation of these processes are ap-



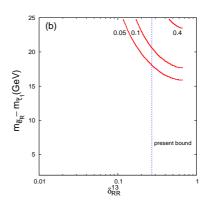
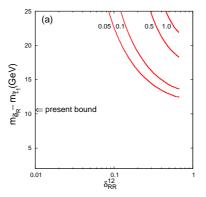


Figure 7: Contours of constant (a)  $\sigma(e^+e^- \to \tilde{e}_R^+\tilde{e}_R^- \to e^+e^-e^\pm\tau^\pm\tilde{\tau}_1^\mp\tilde{\tau}_1^\mp)$  and (b)  $\sigma(e^+e^- \to \tilde{e}_R^+\tilde{e}_R^- \to \tau^+e^-\tau^\pm\tau^\pm\tilde{\tau}_1^\mp\tilde{\tau}_1^\mp)$  and (b)  $\sigma(e^+e^- \to \tilde{e}_R^+\tilde{e}_R^- \to \tau^+e^-\tau^\pm\tau^\pm\tilde{\tau}_1^\mp\tilde{\tau}_1^\mp)$  in fb with  $\sqrt{s}=500\,\mathrm{GeV}$ , and the present experimental upper bound on  $\delta^{13}$  coming from the non-observation of the process  $\tau \to e\gamma$ . The remaining parameters of the model are chosen as in the  $\epsilon$  point (see text for details). Both the  $e^+$  and the  $e^-$  beam are unpolarized.



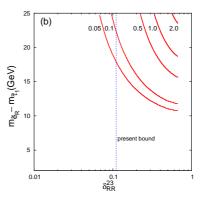


Figure 8: Contours of constant (a)  $\sigma(e^+e^- \to \tilde{e}_R^+ \tilde{e}_R^- \to \mu^+ e^- \tau^\pm \tau^\pm \tilde{\tau}_1^\mp \tilde{\tau}_1^\mp + \mu^- e^+ \tau^\pm \tilde{\tau}_1^\mp \tilde{\tau}_1^\mp)$  and (b)  $\sigma(e^+e^- \to \tilde{e}_R^+ \tilde{e}_R^- \to e^+ e^- \mu^\pm \tau^\pm \tilde{\tau}_1^\mp \tilde{\tau}_1^\mp)$  in fb with  $\sqrt{s} = 500\,\mathrm{GeV}$ , and the present experimental upper bounds on  $\delta^{12}$  and  $\delta^{23}$  coming from the non-observation of the processes  $\mu \to e \gamma$  and  $\tau \to \mu \gamma$ , respectively. Note that the present upper bound for  $\delta^{12}$  lies outside figure (a). The remaining parameters of the model are chosen as in the  $\epsilon$  point (see text for details). Both the  $e^+$  and the  $e^-$  beams are unpolarized.

proximately a factor of five weaker than the constraints coming from processes with pair production of right-handed selectrons, and are therefore poorer probes of lepton flavour violation.

## 4. Conclusions

In this paper we have estimated the sensitivity of future  $e^+e^-$  and  $e^-e^-$  colliders to lepton flavour violation, in scenarios where the gravitino is the lightest supersymmetric particle (LSP) and the stau is the next-to-lightest supersymmetric particle (NLSP). Since the NLSP can only decay gravitationally into gravitinos and charged leptons, the decay rate is very suppressed and the NLSP could traverse several layers of the vertex detector before decaying or even being stopped and trapped in it. This peculiar signature would be a clear signal for this class of scenarios and in particular would allow the clean search for lepton flavour violation, as the Standard Model backgrounds are very small and the supersymmetric backgrounds can be kept under control by using suitable kinematic cuts.

The signals of lepton flavour violation would consist of two heavily ionizing tracks due to the long-lived staus accompanied by two or four charged leptons. Final states with two heavily ionizing tracks and two charged leptons correspond to the lepton flavour violating production of a right-handed selectron and a NLSP, followed by the decay of the righthanded selectron into the NLSP and two charged leptons, and would constitute a signal for lepton flavour violation in the right-handed selectron-stau sector. On the other hand, final states with two heavily ionizing tracks and four charged leptons correspond to the pair production of two right-handed selectrons (and also smuons, in the case of the  $e^+e^$ collider), followed by one lepton flavour violating decay and one lepton flavour conserving decay into the NLSP and two charged leptons. These signals arise when there exists mixing between any two generations of the right-handed sector. Nevertheless, to search for lepton flavour violation in the selectron-stau sector, we find that signals with two charged leptons in the final state are a more sensitive probe than signals with four charged leptons. We also find that the sensitivity to lepton flavour violation is slightly better at the  $e^-e^-$  collider than at the  $e^+e^-$  collider, due to the slightly larger production cross-section of sleptons at the  $e^-e^-$  collider (either in a lepton flavour conserving or in a lepton flavour violating mode), which is due to the constructive interference of the t- and the u-channel production

To illustrate the sensitivity reach of this experiment, we have analyzed in detail a variant of the  $\epsilon$  benchmark point presented in [10], taking all the supersymmetric parameters as in the  $\epsilon$  benchmark point, but varying the NLSP mass between 144 GeV and 167 GeV and admitting some small amount of lepton flavour violation in the right-handed slepton sector, parametrized by  $\delta_{RR}^{ij}$ . We have also estimated the efficiency of detecting the long-lived staus using the traditional methods and folded the efficiency with the calculated signal cross sections.

In particular, when the mass splitting between the NLSP and the next-to-NLSP is  $20\,\mathrm{GeV}$ , we find that the International Linear Collider with a center of mass energy of  $\sqrt{s} = 500\,\mathrm{GeV}$  and an integrated luminosity of  $500\,\mathrm{fb^{-1}}$  could probe lepton flavour violation down to the level  $\delta_{RR}^{13} \sim 0.02$ ,  $\delta_{RR}^{12} \sim 0.04$ ,  $\delta_{RR}^{23} \sim 0.03$  at  $5\sigma$  in the  $e^-e^-$  mode using unpolarized beams, and slightly worse in the  $e^+e^-$  mode. As a side remark we would like to mention that the use of right-polarized electron and positron beams enhances the signal strength and reduces the background cross section. However, the sensitivity to

lepton flavour violation does not improve significantly since the integrated luminosity is also reduced for polarized beams. This sensitivity is competitive with the present and projected sensitivities to lepton flavour violation from rare tau decays, although not from rare muon decays where the non-observation of the process  $\mu \to e \gamma$  still gives the most stringent constraints. Finally, we would like to remark that whereas the origin of the lepton flavour violation cannot be determined just by the observation of rare decays, the observation of the tree level production and/or decay of sleptons at the International Linear Collider would pinpoint the right-handed slepton sector as one of the sources of lepton flavour violation. Complementing this information with the one from rare decays could help to identify the sources of flavour violation in the leptonic sector, providing invaluable information about the soft-breaking Lagrangian.

## A. Approximate formulas for $BR(\ell_i \to \ell_j \gamma)$

We review here the relevant formulas to compute the bounds on the lepton flavour violating parameters  $\delta_{LL}$ ,  $\delta_{RR}$ ,  $\delta_{LR}$ ,  $\delta_{RL}$  from the non-observation of the rare decays  $\ell_i \to \ell_j \gamma$ , following closely the analysis by Masina and Savoy [26]. In the mass insertion approximation, the branching ratio for the process  $\ell_i \to \ell_j \gamma$  reads:

$$BR(\ell_{i} \to \ell_{j}\gamma) = 3.4 \times 10^{-4} BR(\ell_{i} \to \ell_{j}\bar{\nu}_{j}\nu_{i}) \frac{M_{W}^{4}M_{1}^{2} \tan^{2}\beta}{|\mu|^{2}} \times \left\{ \left| \delta_{LL}^{ji}(\eta^{*}I_{B,L}' + \frac{1}{2}I_{L}' + I_{2}') + \delta_{LR}^{ji} \frac{m_{R}^{av}m_{L}^{av}}{\mu m_{i} \tan\beta} I_{B} \right|^{2} + \left| \delta_{RR}^{ji}(\eta I_{B,R}' - I_{R}') + \delta_{RL}^{ji} \frac{m_{R}^{av}m_{L}^{av}}{\mu^{*}m_{i} \tan\beta} I_{B} \right|^{2} \right\},$$
(A.1)

where we have defined

$$\eta = 1 - \frac{A_l^{av}}{\mu^* \tan \beta} ,$$

$$I_B = \frac{1}{m_R^{av2} - m_L^{av2}} \left[ \frac{|\mu|^2}{m_L^{av2}} g_1 \left( \frac{M_1^2}{m_L^{av2}} \right) - \frac{|\mu|^2}{m_R^{av2}} g_1 \left( \frac{M_1^2}{m_R^{av2}} \right) \right] ,$$

$$I'_L = \frac{1}{m_L^{av2}} \frac{|\mu|^2}{|\mu|^2 - M_1^2} \left[ h_1 \left( \frac{M_1^2}{m_L^{av2}} \right) - h_1 \left( \frac{|\mu|^2}{m_L^{av2}} \right) \right] ,$$

$$I'_R = \frac{1}{m_R^{av2}} \frac{|\mu|^2}{|\mu|^2 - M_1^2} \left[ h_1 \left( \frac{M_1^2}{m_R^{av2}} \right) - h_1 \left( \frac{|\mu|^2}{m_R^{av2}} \right) \right] ,$$

$$I'_2 = \frac{M_2 \cot^2 \theta_W}{M_1 m_L^{av2}} \frac{|\mu|^2}{|\mu|^2 - |M_2|^2} \left[ h_2 \left( \frac{|M_2|^2}{m_L^{av2}} \right) - h_2 \left( \frac{|\mu|^2}{m_L^{av2}} \right) \right] ,$$

$$I'_{B,R} = -\frac{1}{m_R^{av2} - m_L^{av2}} \left[ \frac{|\mu|^2}{m_R^{av2}} h_1 \left( \frac{M_1^2}{m_R^{av2}} \right) - m_R^{av2} I_B \right] ,$$

$$I'_{B,L} = -\frac{1}{m_L^{av2} - m_R^{av2}} \left[ \frac{|\mu|^2}{m_L^{av2}} h_1 \left( \frac{M_1^2}{m_L^{av2}} \right) - m_L^{av2} I_B \right] .$$
(A.2)

The functions  $g_{1,2}$  and  $h_{1,2}$  have the following expression:

$$g_1(x) = \frac{1 - x^2 + 2x \ln(x)}{(1 - x)^3},$$

$$h_1(x) = \frac{1 + 4x - 5x^2 + (2x^2 + 4x) \ln(x)}{(1 - x)^4},$$

$$h_2(x) = \frac{7x^2 + 4x - 11 - 2(x^2 + 6x + 2) \ln(x)}{2(x - 1)^4}.$$
(A.3)

In these formulas,  $m_L^{av}$ ,  $m_R^{av}$ ,  $A_l^{av}$  were defined after eq. (1.3),<sup>7</sup>  $m_i$  is the mass of the decaying lepton,  $\theta_W$  is the Weinberg's angle,  $M_1$  is the bino mass (that we choose real, by means of a phase redefinition) and  $M_2$  is the wino mass. The bounds on the  $\delta$  parameters in table 1 can be straightforwardly computed using eq. (A.1), assuming that one single  $\delta$  is the only source of lepton flavour violation.

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<sup>&</sup>lt;sup>7</sup>Note that in contrast to [26] we have absorbed all the flavour dependence of  $A_{l_{ij}}$  in the definitions of  $\delta_{LR,RL}$ .

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